Solutions to Chapter 5 Exercises

Exercise 5.1

See Chapter 5 Exercises.xls for additional solution detail.

(a) $0.008966 = (1-0.008750) \times (0.009045)$
(b) $0.11215 = (1-0.18600) \times (1-0.17969) \times (0.16795)$
(c) $146,228.75

Exercise 5.2

See Chapter 5 Exercises.xls for additional solution detail.

Claim cost for a CMM benefit, with a $1,000 deductible, and 80% coinsurance until $2,000 out-of-pocket expenses occur, after which the benefit is 100% = $2,270.69

Exercise 5.3

See Chapter 5 Exercises.xls for additional solution detail.

After 10% inflation, the claim cost for a CMM benefit, with a $1,000 deductible, and 80% coinsurance until $2,000 out-of-pocket expenses occur, after which the benefit is 100% = $2,545.28

Claim cost inflation = 10%
Leveraged inflation = $2,545.28/$2,270.69 − 1 = 12%

The leveraged inflation is the inflation on the amount paid by the insurer.

Exercise 5.4

Assumes all months have 30 days.

(1) Premium Received

Let X = date the premium was received
X~ Uniform (0, 1), where $X = 0 =$ January 1 and $X = 1 =$ December 31
$E(X) = (0+1)/2 = 0.5$
Weighted average date of premium received in months = 12(0.5) = 6
Weighted average date of premium received = July 1

(2) Premium Earned

Let X = date the premium was received
X ~ Uniform (0, 1), where $X = 0 =$ January 1 and $X = 1 =$ December 31
Let $Y =$ date of the midpoint for the period between X and December 31
$Y = (X+1)/2$

$$E(Y) = E\left[\frac{X+1}{2}\right] = \frac{1}{2}\int_{x=0}^{1}(x+1)\,dx = \frac{1}{2}\left[\frac{x^2}{2}+x\right]_{x=0}^{1} = \frac{1}{2}\left[\frac{1}{2}+1\right] = 0.75$$

Weighted average date of premium earned in months = 12(0.75) = 9

Weighted average date of premium earned = October 1

(3) Claims Incurred

See Chapter 5 Exercises.xls for additional solution detail.

Weighted average claim incurral date = July 5

(4) Claims Paid

See Chapter 5 Exercises.xls for additional solution detail.

Weighted average claim payment date = August 3

Exercise 5.5

Assumes all months have 30 days.

(1) Premium Received

$Z =$ random variable representing the incurral month, where Jan = 1 … Dec = 12

$X =$ random variable representing the date of claim incurral

$Z \sim$ Discrete Uniform (12), where $p(Z=z) = 1/12$ for $1 \le z \le 12$

$X \mid Z \sim$ Uniform (a, b)

$$E[X] = E\left[E[X \mid Z]\right] = \sum_z \left\{E[X \mid Z=z] \times p(X=z)\right\}$$

See Chapter 5 Exercises.xls for additional solution detail.

Weighted average date of premium received = July 1

(2) Premium Earned

$Y \mid Z =$ date of the average exposure of the premium earned during the calendar year, given month z

For $1 \le z \le 11$, all premium has been earned during the calendar year (not necessarily during month z), therefore $y =$ end of month z

For example (assuming equal days per month, i.e., 30):

$x1 = 1/01$, premium 100% earned by $2/01 >> y1 = 1/15$

$x2 = 1/02$, premium 100% earned by $2/01 >> y2 = 1/16$

$\vdots$

$x30 = 1/30$, premium 100% earned by $2/30 >> y30 = 2/14$

Therefore, $E[Y \mid \text{January}] = 1/30$, the end of month $z =$ January.

For $z = 12$, not all of the premium will be earned during the calendar year, each policy issued average exposure is halfway to the end of the calendar year.

$$E(Y \mid \text{December}) = E\left[\frac{X+1}{2} \,\middle|\, \text{December}\right] = \int_{x=0.9167}^{1} \frac{x+1}{2} \times \left(\frac{1}{1-0.9167}\right) dx$$

$$= \left(\frac{1}{2}\right) \times (12) \times \left[\frac{x^2}{2} + x\right]_{x=0.9167}^{1}$$

$$= \frac{1}{2}\left[\left(\frac{1^2}{2} + 1\right) - \left(\frac{0.9167^2}{2} + 0.9167\right)\right] = 0.9792,$$

this corresponds to 11.75 months or Dec 23

$$E[Y] = E\left[E[Y \mid Z]\right] = \sum\nolimits_z \left\{E[Y \mid Z=z] \times p(Z=z)\right\}$$

See Chapter 5 Exercises.xls for additional solution detail.
Weighted average date of premium earned = July 15

(3) Claims Incurred

Changing from an annual premium mode to a monthly premium mode, does not affect the claims.
The results are the same as Exercise 5.3 (3).
Weighted average claim incurral date = July 5

(4) Claims Paid

Changing from an annual premium mode to a monthly premium mode, does not affect the claims.
The results are the same as Exercise 5.3 (3).
Weighted average claim payment date = August 3